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BOOK REVIEW

Worlds of Flow: A History of Hydrodynamics from the Bernoullis to Prandtl. By O. Darrigol. Oxford University Press, 2005. 356 pp. ISBN 0 19 856843 6. £35.00 (hardback)

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Fluid mechanics has attracted great scientists and mathematicians throughout history, and among other flow topics, turbulence remains one of the last great unsolved problems from the era of classical mechanics. The satisfaction of formulating mathematical flow models (and trying to find their solutions) has long been frustrated by the difficulty of relating the predictions to observations of nature or measurements of flows under laboratory conditions. The recurring disagreements between beautiful mathematical theories and the beguiling complexities of real fluid flows have made the history of hydrodynamics a stop–go story of false starts, reverses and occasional forward jumps. Sophisticated theories have been invented, then followed up by algebraic simplifications and clarifications, only for the whole process to be repeated independently elsewhere. Favoured theories were repeatedly contradicted by observations; and the observed phenomena have often remained stubbornly unexplained, sometimes up to today. Indeed the refinement of flash and high-speed photography over the past century, for example, has posed new challenges for theorists.

Professional historians of science have paid surprisingly little attention to fluid mechanics. Two monumental histories of mechanics, one by E. Mach and the other by R. Dugas, only give a few pages to hydrodynamics, and then only include hydrostatics and quasi-one-dimensional flows. It may be that the technical difficulties are a deterrent to outsiders, but there have always been many engineers and scientists who have worked on flow problems of commercial, military and environmental importance. So the growth of fluid mechanics, and its influence on modern civilization, are serious parts of the history of ideas. Our subject's web of connected ideas deserves to have its historic threads respun as a story about the researchers and their affects on each other. Olivier Darrigol's book weaves this narrative admirably.

What other works on the history of hydrodynamics are there? One of the few general books is *History of Hydraulics* by H. Rouse & S. Ince, Dover, 1957. This covers the ground from ancient times to the mid-twentieth century, and examines the key personalities, with fine portraits and illustrations drawn from the primary literature. To my knowledge, the only other history of fluid mechanics, in book form, is by G. A. Tokaty and published by Dover in 1994, which is so idiosyncratic that I will pass on.

This work is therefore a welcome addition to scholarship on the history of fluid flow science. I have not seen Olivier Darrigol's work before, but the back flap describes him as 'Research Director at the Centre National de Recherche Scientifique, Paris'. He has published papers in journals of scientific history. In the book he describes well the problems of fluid mechanics, and their resolution, with modern understanding, as well as showing us the viewpoints of the discoverers. The verbal or limited mathematical explanations which one finds in historic sources, Darriogol helpfully re-explains in modern analytic terms which should pose no difficulty to a final-year undergraduate who has studied hydrodynamics.

The subtitle of *Worlds of Flow* defines a limited period of time: from the publication in 1738 of Daniel Bernoulli's famous equation, through to the 1904 conference paper (proceedings published in 1905) of Ludwig Prandtl, on the concept of the boundary layer. This is a reasonable date at which to end the book because Prandtl's theory of the boundary layer marks the end of a long era of confusion about drag, and it defines a start date for the modern era of fluid mechanics. The confusion about drag began in 1752 when D'Alembert identified a so-called paradox: that in the steady unbounded inviscid flow past a fixed body (with no circulation round the body), the theory predicts a zero drag, at odds with real fluid flows. The confusion surrounding the applicability of inviscid flow theory was compounded by the discoveries, in the nineteenth century, of free-surface wave drag, and time-dependent forces (associated with unsteady body motions in the flow). After 1904 Prandtl and his coworkers developed boundary-layer theory in ways stimulated by the early years of powered flight. Prandtl's 1904 paper marks the start of a still ongoing and active field of science, and Darrigol accounts for some of Prandtl's later career. (For a review of the more recent fluid mechanics, post Prandtl, see Sir James Lighthill's description in volume II of *Twentieth Century Physics*, edited by L. M. Brown, A. Pais & B. Pippard, pp. 795–912, published by the Institute of Physics, 1997. I am grateful to Norman Riley for this reference.)

Some topics are understandably absent: Darrigol declares his intention of ignoring the thermodynamics of fluid motion. This rules out the developments in steam power, balloon flight, naval and military ballistics, and internal combustion engines. There could have been more about the history of capillarity and the work on thin films conducted by Joseph Plateau (but this is covered by C. Isenberg, *The Science of Soap Films and Soap Bubbles*, published by Dover in 1992). In his book Darrigol keeps to his declared aims and wisely ignores the prehistory of fluid mechanics, before about 1680. Other writers on this prehistory dwell on topics such as hydrostatics, pneumatics, water supply and the speculations noted and drawn by Leonardo Da Vinci [1452–1519]. On the work of Da Vinci see C. Truesdell *Essays in the History of Mechanics*, Elsevier, 1968. Another sadly ignored work, which focuses on this early period, is by P. F. Neményi 'The main concepts and ideas of fluid dynamics in their historical development', *Archive for the History of the Exact Sciences*, vol. 2 (1962), pp. 52–86.

Despite starting in about 1738, Darrigol does throw light on some significant moments dating from the earlier years of fluid dynamics, before Daniel Bernoulli, for instance the influential contributions from Isaac Newton. A few years ago it was a surprise to me to learn that Book II of Newton's *Principia* (1687) contains much hydrodynamics. Newton wanted to establish enough fluid mechanics to crush Descartes' vortex theory of planetary motion. (Descartes mistakenly believed that a fluid force, supplied by a rotating flow, was needed to sustain the planets in their orbits about the sun.) As Darrigol explains, Newton's impact theory of the drag on a bluff body, such as a spherical planet, was spoiled by unconvincing explanations. Likewise his pioneering measurements of the *vena contracta* are explained strangely. Under the weight of Newton's authority, researchers of the 18th century had to work hard to overturn some misleading ideas in the *Principia*. For example, Newton correctly treated the oscillations of a thread of water, of length λ, in a U-tube. I looked up the relevant passages, starting at Proposition 44 Theorem 35, in I. B. Cohen & A. Whitman's new translation of the *Principia*, University of California Press. Newton's reasoning contains no equations. He starts with what I think are deep-water *standing* waves and infers, somehow, that the period of a water wave of length λ is directly proportional to $\lambda^{1/2}$. This is correct but the constant of proportionality that Newton describes makes the wave period about 25% too long. Newton says that 'the time has been determined only approximately'. He notes his own inconsistency in that the fluid particles in *travelling* waves move in circular orbits, not up and down as in the U-tube (or as occurs beneath the crest and trough of a *standing* wave!). Laplace criticized Newton's lack of clarity in a paper of 1776. In that year Laplace treated standing waves in deep water and came within an ace of formulating expressions for travelling waves; a gap which Lagrange filled five years later. This is just one example of the many threads of thought and influence which Darrigol traces through the history of fluid dynamics.

Chapter 2 treats the history of water wave theory and experiments. I learnt much, for example, from the story of the initial value problem, which progressed from complex mathematical memoirs by Cauchy and Poisson in the 1820s, through to the accounts of G. G. Stokes, Lord Kelvin (William Thomson), Lord Rayleigh and Horace Lamb. In 1932 Lamb updated his *Hydrodynamics* for a sixth edition, still much cited by *JFM* authors, but his book first appeared (as *Treatise on the Mathematical Theory of the Motion of Fluids*) 53 years earlier in 1879.

Darrigol achieves a good balance between the competing temptations to use space for biographical anecdotes, mathematical expositions and illustrations (here crisply printed in black and white). In Chapter 2 he describes the career of John Scott Russell and his observations of solitary water waves, from 1835 onward. Russell was criticized by Airy in 1845 and by Stokes, who for forty years did not believe that the solitary wave could be a solution of the *steady* equations of inviscid flow. Lord Rayleigh in 1870 (and Boussinesq in 1871) obtained theories of the solitary water wave. This was all before Korteweg and de Vries's much cited paper of 1895 which, incidentally, includes surface tension. Another historical oddity is the discovery that capillarity allows arcs of water-wave crests to form *upstream* of a fishing line held in a steady flow. This was known to (and drawn in a beautiful plate by) Russell in 1845, decades before Lord Kelvin made experiments in 1871 on capillary waves, from his yacht (with Helmholtz as an on-board guest). Nowadays the credit goes to Rayleigh, who overcame the then formidable mathematical difficulties needed to account for the steady surface wave pattern, in 1883.

The eventual successes of inviscid fluid mechanics in modelling water waves benefited ship design and led to Kelvin's ship-wave-wake theory. These successes are contrasted, in later chapters, with the nineteenth century failure to reconcile approximate treatments of the Navier–Stokes equations with the real fluid flows measured by engineers in water supplies, canals, drainage systems and rivers. (An exception is Reynolds's lubrication theory.) The scientific struggle with the influence of viscosity is a wonderful tale. Chapter 3 tells the story of the many derivations of the equations of viscous flow, from Navier in 1822, through Cauchy in 1828, and Saint-Venant in 1843, to Stokes in 1845. In contrast with Navier, Stokes advocated a zero fluid velocity as the correct boundary condition at a fixed impermeable surface.

Vortices and Hermann Helmholtz's 1860s study of the motion of a system of vortices are treated in Chapter 4. Helmholtz's work on the acoustics of organ pipes showed that in sound production the role of vortices is crucial to inducing pressure fluctuations. Helmholtz showed that the viscosity of the air vibrating in the pipe alters the frequencies away from those resonances predicted by inviscid theory. Helmholtz found that the optimal width-to-length ratio of pipes agreed with a rule of thumb known a century before to the master organ builder Andreas Silbermann. (In the 1730s

Silbermann's brother, Gottfried, often invited his friend J. S. Bach [1685–1750] to test the family firm's new musical instruments, and it was Bach who took a mathematical view of equal temperament for musical frequencies.) Darrigol also reflects in this chapter on the role of the Earth's rotation and vorticity in meteorology, with Helmholtz's early explanations for the Föhn (hot dry winds in mountain terrain) and the causes and movements of tornadoes.

We move on to the general problems of flow instability in Chapter 5. Kelvin and Rayleigh maintained an amiable scientific correspondence for decades. They argued back and forth, for example, about the formation and stability of surfaces of velocity discontinuity. Stokes took the side of Helmholtz and his model for their formation, and he thought that this was the key to understanding drag. But Kelvin remained sure that such surfaces would be unstable for a fluid with non-zero viscosity. Apart from other aspects of flow instability, Darrigol gives examples of the help that researchers gave each other. In 1868, at the start of a climbing holiday, Karl Weierstrass gave Hermann Helmholtz a copy of Bernhard Riemann's PhD thesis, from which Helmholtz thought of using complex variables to represent streamlines in two-dimensional flow. (Few noticed that in 1752 D'Alembert had already made this connection in print.) The subsequent work of Helmholtz, Kirchoff and Rayleigh built a body of knowledge, including the conformal mappings which quickly featured in university hydrodynamics exam papers.

The sixth chapter, on turbulence, brings us back to the preoccupations of the Victorian hydraulics community. This was the age of great civil engineering projects: canal building, and city water supplies. Darrigol traces the master–student collaborations of the theorists from the 1820s onward, e.g. Navier, Cauchy, Poisson, Saint-Venant and Boussinesq in their works on open-channel flows. The chapter culminates in the dimensional analysis and experiments of Osborne Reynolds. In pipe-flow experiments at Manchester (dating from 1876), reported in 1883, Reynolds reported the transition from laminar to turbulent motion. He had been anticipated, in 1839, by the German hydraulician Gotthilf Hagen, who demonstrated a transition to unstable pipe flow. But it was Reynolds who deserves the credit for scaling the Navier–Stokes equations to a dependence on just one dimensionless number, a critical value of which he found for his apparatus at Manchester. Reynolds also reported observations which remain of theoretical interest today. His 1883 paper (*Phil. Trans. R. Soc. Lond.*, vol. 174, p. 935) is worth rereading.

Very few non-Europeans feature in this book. But it was a pair of Americans who first built and flew a successful aircraft. Darrigol's last main chapter describes the analysis of drag and lift, for flows past bodies which possess circulation. The success of this modelling gave aerodynamicists a tentative theory for flight even before the Wright brothers first took off in 1903. The work on aerodynamics by Frederick Lanchester, published in 1907–8, influenced the growth of Ludwig Prandtl's ideas after his 1904 boundary-layer paper. In fact the self-educated Lanchester already had a shrewd qualitative account of the production and transport (in three dimensions) of vorticity next to wing surfaces. Darrigol nicely expounds the mathematical reasoning which Prandtl and others used, mainly during the First World War, to work up ideas into practical methods of calculating flows past real aircraft wings. This at last brought hydrodynamic theory into agreement with aerodynamical measurements of lift and drag in controlled flight.

The author has translated quotations from the many primary sources that he cites. The bibliography lists original works alongside many useful secondary sources. In the footnotes the precision about source material adds to this scholarly work. But this is not a dry book: judge for yourself a quote that Darrigol takes from Theodore von Kármán who was writing about Prandtl, his former PhD supervisor:

> 'I came to realize that ever since I came to Aachen my old professor and I were in a kind of world competition . . . a kind of Olympic games, between Prandtl and me, and beyond that between Göttingen and Aachen. The 'playing field' was the Congress of Applied Mechanics and the 'ball' was the search for a universal law of turbulence'.

I also enjoyed Darrigol's account of a meeting at Le Mans in 1908, when Frederick Lanchester went to see the Wright brothers' machine at its first flight in Europe:

> 'Lanchester found Wilbur Wright very ill-disposed toward theory. The pioneering constructor dryly commented that the most talkative bird (the parrot) is also a poor flier.'

One example of the pointed force of Darrigol's style is his description of Franz Joseph von Gerstner in 1802 as a '. . . Prague professor of mathematics, engineer and knight'.

I look forward to a paperback edition of this book, with some typos corrected, and published at a price low enough for fluid dynamicists around the world to afford. Meanwhile ask your library to buy it, and share this pleasure with others. I found Olivier Darrigol's *Worlds of Flow* a very enjoyable book. Reading it will reward all those who care about the problems of fluid mechanics.

M. J. Cooker